Course Syllabus

I. General Information

Course name	Elements of number theory
Programme	mathematics
Level of studies (BA, BSc, MA, MSc, long-cycle MA)	BA
Form of studies (full-time, part-time)	full-time
Discipline	mathematics
Language of instruction	english

Course coordinator/person responsible	dr Wiesław Główczyński

Type of class *(use only the types mentioned below)*	Number of teaching hours	Semester	ECTS Points
lecture	30	III, V	5
tutorial			
classes	30	III, V	
laboratory classes			
workshops			
seminar			
introductory seminar			
foreign language classes			
practical placement			
field work			
diploma laboratory			
translation classes			
study visit			

Course pre-requisites	Introduction to mathematics

II. Course Objectives

To present a rigorous development of elementary number theory using axioms, definitions, examples, theorems and their proofs.

III. Course learning outcomes with reference to programme learning outcomes

Symbol	Description of course learning outcome	Reference to programme learning outcome
KNOWLEDGE		
W_01	The student understands the importance of mathematics and its applications, in particular its role in the context of contemporary civilization's dilemmas	K_W01
W_02	The student has advanced knowledge of the basic areas of higher mathematics, in particular in calculus, algebra, geometry, logic, measure and integral, probability theory, differential equations, statistics, set theory, topology and others selected fields of mathematics and its applications.	K_W04
SKILLS		
U_01	The student is able to use his knowledge to formulate complex and unusual mathematical problems in a correct and understandable way, discuss them and methods of solving them and present mathematical results and contents, in particular using information and communication techniques	K_U38
SOCIAL COMPETENCIES		
K_01	The student is prepared to appreciate the role and importance of knowledge in solving cognitive and practical problems, typical of occupations and workplaces appropriate for graduates in the field of mathematics and consulting experts in the case of difficulties in solving the problem	K_K02
K_02	The student is ready to present selected achievements of higher mathematics in a popular way	K_K05

IV. Course Content

1.Divisibility Theory in the Integers (The Division Algorithm,The Greatest Common Divisor, The Euclidean Algorithm, The Diophantine Equation ax + by = c).

2. Primes and Their Distribution (The Fundamental Theorem of Arithmetic. The Sieve of Eratosthenes ,The Goldbach Conjecture, Bertrand's Postulate).

3. The Theory of Congruences (Basic Properties of Congruence,Binary and Decimal Representations of Integers, Linear Congruences and the Chinese Remainder Theorem).

4. Fermat's Theorem (Fermat's Little Theorem and Pseudoprimes, Wilson's Theorem, The Fermat-Kraitchik Factorization Method).

5 Number-Theoretic Functions (The Sum and Number of Divisors,The Greatest Integer Function)

6. Euler's Generalization of Fermat's Theorem (Euler's Phi-Function, Euler's Theorem, Properties of the Phi-Function).

7. Primitive Roots and Indices (The Order of an Integer Modulo n, Primitive Roots for Primes, Composite Numbers Having Primitive Roots,The Theory of Indices).

8.The Quadratic Reciprocity Law (Euler's Criterion, The Legendre Symbol and Its Properties, Quadratic Reciprocity, Quadratic Congruences with Composite Moduli).

9. Numbers of Special Form (Perfect Numbers, Mersenne Primes and Amicable Numbers Fermat Numbers)
10. Certain Nonlinear Diophantine Equations (The Equation $x^2 + y^2 = z^2$, Fermat's Last Theorem).
12. Representation of Integers as Sums of Squares (Sums of Two Squares, Sums of More Than Two Squares).
13. Fibonacci Numbers,The Fibonacci Sequence, Certain Identities Involving Fibonacci Numbers.
14. Open problems in Elementary Number Theory.

V. Didactic methods used and forms of assessment of learning outcomes

Symbol	Didactic methods *(choose from the list)*	Forms of assessment *(choose from the list)*	Documentation type *(choose from the list)*
KNOWLEDGE			
W_01	Conventional lecture/ Practical classes	Exam / Written test	Protocol
W_02	Conventional lecture/ Practical classes	Exam / Written test	Protocol
SKILLS			
U_01	Conventional lecture/ Practical classes	Exam / Written test	Protocol
SOCIAL COMPETENCIES			
K_01	Conventional lecture/ Practical classes	Exam / Written test	Protocol
K_02	Conventional lecture/ Practical classes	Exam / Written test	Protocol

VI. Grading criteria, weighting factors.....

Exam (for students who passed classes):

– in groups of less than 8 students – oral exam

– in groups of 8 or more students – written exam (and oral exam for students who didn't received 50% points at written exam).

Exam, passing level is 50% of the sum of points;

91% – 100% excellent (5.0)

81% – 90% very good (4.5)

71% – 80% good (4.0)

61% – 70% satisfactory (3.5)

50% – 60% sufficient (3.0)

less than 50% fail (2.0)

W1, W2 - discussion on lessons, colloquium, exam

U1 - discussion on lessons, colloquium, exam

K1, K2 - discussion on lessons

In groups of less than 8 students credits are given by active participation in classes.

Colloquium, passing level is 50% of the sum of points;

91% – 100% excellent (5.0)

81% – 90% very good (4.5)

71% – 80% good (4.0)

61% – 70% satisfactory (3.5)

50% – 60% sufficient (3.0)

less than 50% fail and lack of active participation in classes (2.0)

W1, W2 - discussion on lessons, colloquium,

U1 - discussion on lessons, colloquium,

K1, K2 - discussion on lessons

Hourly equivalent to ECTS credits:

Lecture - 30

Classes - 30

Consultations - 30

Preparation for classes including self-solving of tasks identified by the teacher - 30

Preparing for the tests and exam, including reading the literature - 30

Total number of hours 150.

Total number of ECTS credits per module 5

VII. Student workload

Form of activity	Number of hours
Number of contact hours (with the teacher)	90
Number of hours of individual student work	60

VIII. Literature

Basic literature
1. D. M. Burton Elementary Number Theory, Seventh Edition, McGraw Hill (2011). 2. J. A. Anderson, J. M. Bell, Number Theory with Applications, Pearson (1996). 3. T. Koshy, Elementary Number Theory with Applications, Academic Press (Second Edition (2007) 4. T. Koshy, Student Solutions Manual for Elementary Number Theory with Application, Academic Press (2002). 5. J. H. Silverman, A Friendly Introduction to Number Theory, 4th Edition, Pearson (2011). 6. J. H. Silverman, Instructor's Solutions Manual for A Friendly Introduction to Number Theory -4th Edition Pearson (2013).
Additional literature
1.K. Rosen, Elementary Number Theory and its Applications, Pearson/Addison-Wesley (Fifth edition, 2005). 2. B. Goddard, Kenneth H. Rosen Student's Solutions Manual Elementary Number Theory and its Applications - Fifth edition, Pearson (2005). 3. W. Stein, Elementary Number Theory: Primes, Congruences, and Secrets, A Computational Approach, Springer (2009). 4. R. Kumanduri, C.Romero Number Theory with computer applications, Prentice Hall (1998). 5. N. R. Reilly, Introduction to Applied Algebraic Systems, Oxford University Press (2008). 6. W. Sierpinski, Elementary Theory of Numbers, North Holland, Second English Edition (1988). 7. W. Sierpinski, 250 Problems in Elementary Number Theory, PWN (1970). 8. D. Shanks, Solved and Unsolved Problems in Number Theory, AMS (2001).